1. Suppose $\{x_n\}_{n=1}^{\infty}$ and $\{y_n\}_{n=1}^{\infty}$ are such that for every $\epsilon > 0$ there is an M such that $| x_n - y_n |< \epsilon$ for all $n \geq M$. If $x_n \to x$ then does it imply that y_n converges.

2. Let $x \in \mathbb{R}$. Decide for what all $x \in \mathbb{R}$ do the following two power series converge:

$$\text{(a)} \quad \sum_{n=1}^{\infty} \frac{3^n}{1+\frac{3}{n}} x^n \quad \text{and} \quad \text{(b)} \quad \sum_{n=1}^{\infty} \frac{10}{\sqrt[3]{n}} x^n$$

3. Decide (giving adequate justification via a proof or counter-example) whether the following statements are true or false:-

(a) Let $c \in \mathbb{R}$, $\{a_n\}_{n\geq 1}$ be a sequence of real numbers and let $b_n = ca_n$. Then

$$\limsup_{n\to\infty} b_n = c \limsup_{n\to\infty} a_n$$

(b) Let $\alpha > 1$. Let $f : [0, \infty) \to \mathbb{R}$ be given by $f(x) = x^{\alpha} \sin(\frac{1}{x})$. Then f is differentiable at 0.

(c) Let $f : \mathbb{R} \to \mathbb{R}$ be a function. Define $g : \mathbb{R} \to \mathbb{R}$ by $g(x) = \max(0, f(x))$. If g is continuous then f is continuous.

(d) Let $f : \mathbb{R} \to \mathbb{R}$ be a bounded function. Then

$$\sup\{f(x^3) : x \in \mathbb{R}\} = \sup\{f(x) : x \in \mathbb{R}\}$$

(e) Let $a \in \mathbb{R}$ be a limit point of a set $A \subset \mathbb{R}$. Let $\epsilon > 0$. Then $A \cap (a - \epsilon, a + \epsilon)$ is always uncountable.

4. Let $f : \mathbb{R} \to \mathbb{R}$ and suppose that

$$| f(x) - f(y) |\leq (x - y)^2$$

Show that f is a constant function.

5. Let $f : (0, \infty) \to \mathbb{R}$ be a function given by $f(x) = e^x$. It is known that the f is differentiable at all $x > 0$ and $f'(x) = f(x)$ for all $x > 0$. Show that

$$e^{-x} < 1 - x + \frac{x^2}{2}$$

for all $x > 0$.